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# DETERMINATION OF THE STABILITY AND FRAGMENTATION LENGTH OF A MELT JET IN WATER

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The fragmentation of a high-temperature melt jet in water is one of the principal mechanisms underlying the formation of a coarsely dispersed water–steam–melt mixture in the onset and development of a hypothetical severe nuclear power plant accident with core meltdown. Under certain conditions the melt can mix explosively with the water in such a mixture with potentially detrimental results for the reactor housing [1, 2]. The rate of fragmentation of a melt jet largely governs the characteristics of the resulting mixture and its capacity to detonate.

Theofanous and Saito [3] have generated images of the fragmentation process of a melt jet in water and have shown that losses of hydrodynamic stability of the jet flow is a possible fragmentation mechanism. The authors also note that the low density of the steam film formed near the surface of the high-temperature jet can limit the fragmentation rate. The stability of a melt jet in water has been analyzed [4] in a planar setting and in the ideal fluid approximation. It is concluded in [4] that the mixing of an appreciable fraction of the melt with water is hindered by the shielding action of the thick film of steam separating the melt and the water.

In the present article we generalize the work of [4] to a cylindrical geometry, departing from [4] in that we obtain a more realistic estimate of the thickness of the steam film. We compare the analytical results with experimental data on the fragmentation of a molten aluminum jet in water.

**1.** We interpret the basic unperturbed state as the flow of a cylindrical melt jet with constant velocity $U_1$ directed along the z axis (see Fig. 1). A cylindrical steam film having an inside radius $a$ and an outside radius b separates the jet from an unbounded water reservoir. The steam and water move with constant velocities $U_2$ and $U_3$ parallel to $U_1$. The densities of the melt, the steam, and the water are $\rho_1$, $\rho_2$, and $\rho_3$. The subscripts 1, 2, and 3 are used everywhere to designate the melt, steam, and water, respectively. We regard all the media as ideal fluids.

We assume that small, harmonic, axisymmetric disturbances accumulate at the melt–steam ($r_{12}$) and steam–water ($r_{23}$) interfaces:

$$r_{12} = a + \eta_0\exp(ikz - i\omega t),\ r_{23} = b + \xi_0\exp(ikz - i\omega t).$$

Here $\eta_0$ and $\xi_0$ are unknown constants, k and $\omega$ are the wave number and angular frequency of the superimposed disturbances, z is the axial coordinate, and t is the time. We express the resulting small velocity perturbations in terms of the velocity potential:

$$u = \partial\varphi/\partial z,\ v = \partial\varphi/\partial r$$

($\varphi$ is the velocity potential, u and v are the perturbations of the axial and radial velocity components, and r is the radial coordinate). The velocity potential obeys the Laplace equation

$$\frac{\partial^2\varphi_j}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi_j}{\partial r} + \frac{\partial^2\varphi_j}{\partial z^2} = 0\ (j = 1, 2, 3). \tag{1.1}$$

We seek a solution of Eq. (1.1) in the form

$$\varphi_j = f_j(r)\exp(ikz - i\omega t)\ (j = 1, 2, 3). \tag{1.2}$$

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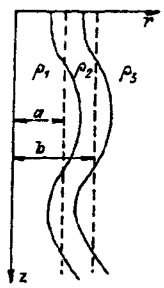


Fig. 1

Substituting Eq. (1.2) into (1.1), we obtain

$$f_j'' + f_j'/r - k^2 f_j = 0 \ (j=1, 2, 3). \tag{1.3}$$

The final solutions of Eq. (1.3) for the zones occupied by the melt, the steam, and the water have the form

$$f_1 = A_1 I_0(kr), \ f_3 = A_3 K_0(kr),$$
$$f_2 = A_2 I_0(kr) + B_2 K_0(kr),$$

where $I_0$ and $K_0$ are zero-th-order Bessel functions of an imaginary argument, and $A_1$, $A_2$, $A_3$, and $B_2$ are unknown constants.

For the perturbed flow the pressure is determined from the linearized Lagrange–Cauchy integral

$$P_j = -\rho_j \frac{\partial \varphi_j}{\partial t} - \rho_j U_j \frac{\partial \varphi_j}{\partial z} + P_j^0 \ (j = 1, 2, 3)$$

($P_j^0$ is the pressure in the case of the unperturbed flow).

**2.** The following dynamical and kinematic conditions must be satisfied at the interfaces of the media.

The pressure difference must be equalized by surface tension at the inner and outer surfaces of the steam film:

$$r=a\colon P_1 = P_2 + \frac{\sigma_{12}}{a} - \frac{\sigma_{12}}{a^2}\left[\eta + a^2 \frac{\partial^2 \eta}{\partial z^2}\right]; \tag{2.1}$$

$$r = b\colon P_2 = P_3 + \frac{\sigma_{23}}{b} - \frac{\sigma_{23}}{b^2}\left[\xi + b^2 \frac{\partial^2 \xi}{\partial z^2}\right] \tag{2.2}$$

($\sigma_{12}$ and $\sigma_{23}$ are the coefficients of surface tension of the melt and the water relative to the steam).

Conditions (2.1) and (2.2) need to be augmented with the four kinematic conditions

$$r = a\colon v_j = \frac{\partial \eta}{\partial t} + U_j \frac{\partial \eta}{\partial z} \ (j=1, 2); \tag{2.3}$$

$$r = b\colon v_k = \frac{\partial \xi}{\partial t} + U_k \frac{\partial \xi}{\partial z} \ (k = 2, 3). \tag{2.4}$$

Substituting the solutions into conditions (2.1)–(2.4), we obtain a linear homogeneous system in the constants $A_1$, $A_2$, $A_3$, $B_2$, $\eta_0$, and $\xi_0$:

$$-i\rho_1(\omega - kU_1)I_0(a)A_1 + i\rho_2(\omega - kU_2)K_0(a)B_2$$
$$+ i\rho_2(\omega - kU_2)I_0(a)A_2 + \sigma_{12}(k^2 - a^{-2})\eta_0 = 0,$$
$$- i\rho_2(\omega - kU_2)I_0(b)A_2 + i\rho_2(\omega - kU_2)K_0(b)B_2$$
$$+ i\rho_3(\omega - kU_3)K_0(b)A_3 + \sigma_{23}(k^2 - b^{-2})\xi_0 = 0,$$
$$kI_0'(a)A_1 + i(\omega - kU_1)\eta_0 = 0,$$
$$kI_0'(a)A_2 + kK_0'(a)B_2 + i(\omega - kU_2)\eta_0 = 0,$$
$$kI_0'(b)A_2 + kK_0'(b)B_2 + i(\omega - kU_2)\xi_0 = 0,$$
$$kK_0'(b)A_3 + i(\omega - kU_3)\xi_0 = 0.$$

TABLE 1

| $\beta$ | $x_m$ | $t_m^{-1}$ | $L/2a$ |
|---|---|---|---|
| 1 | 36,47 | 154,48 | 3,03 |
| 1,001 | 3,10 | 5,557 | 7,16 |
| 1,006 | 1,48 | 1,703 | 11,16 |
| 1,01 | 1,38 | 1,217 | 14,57 |
| 1,1 | 0,82 | 0,4278 | 24,62 |
| 2 | 0,706 | 0,3507 | 25,86 |
| 4 | 0,704 | 0,3475 | 26,03 |
| ∞ | 0,704 | 0,3473 | 26,04 |

TABLE 2

| Experiment No. | $a$, cm | $U_1$, m/sec | We | $T_1$, K | $L/2a$ (experiment) | $L/2a$ (cal.) |
|---|---|---|---|---|---|---|
| 9 | 0,5 | 2,5 | 82,5 | 973 | 9 | 7,51 |
| 10 | 1 | 2,5 | 165 | 973 | 10 | 7,41 |

The Bessel functions are differentiated with respect to the complete argument; from now on we drop the wave number k from the arguments of the Bessel functions for brevity.

The given system has a nontrivial solution only when the determinant is equal to zero. This condition yields the dispersion relation

$$\begin{aligned}&[I_{01}(a)\rho_1(\omega - kU_1)^2 - \sigma_{12}k(k^2 - a^{-2})]\{K_{01}(b)\,[H_1(a)\\&+ H_0(b)]\rho_2(\omega - kU_2)^2 + [K_{01}(b)\rho_3(\omega - kU_3)^2\\&- \sigma_{23}k(k^2 - b^{-2})]\,[H_1(b) - H_1(a)]\} + K_{01}(a)\rho_2(\omega\\&- kU_2)^2\{K_{01}(b)\,[H_0(b) - H_0(a)]\rho_2(\omega - kU_2)^2\\&+ [K_{01}(b)\rho_3(\omega - kU_3)^2 - \sigma_{23}k(k^2 - b^{-2})]\,[H_0(a) + H_1(b)]\} = 0,\end{aligned} \tag{2.5}$$

where $I_{01}(x) = I_0(x)/I_1(x)$, $K_{01}(x) = K_0(x)/K_1(x)$, $H_0(x) = I_0(x)/K_0(x)$, $H_1(x) = I_1(x)/K_1(x)$, and $I_1(x)$ and $K_1(x)$ are first-order Bessel functions of an imaginary argument.

Similar dispersion relations have been obtained in earlier papers [5, 6], which are close in context.

**3.** Inasmuch as the dispersion relation (2.5) is very cumbersome in its general form, we begin with several limiting cases involving major simplifications of the relation.

**Transition to the Planar Problem.** Let the radius of the jet become infinite ($ka \to \infty$, $kb \to \infty$). The Bessel functions then admit the asymptotic representations

$$x \to \infty:\ I_0(x) \approx I_1(x) \approx \exp(x)/\sqrt{2\pi x}; \tag{3.1}$$

$$x \to \infty:\ K_0(x) \approx K_1(x) \approx \exp(-x)/\sqrt{2\pi x}. \tag{3.2}$$

In light of expressions (3.1) and (3.2), Eq. (2.5) acquires the form

$$\begin{aligned}&\mathrm{th}(k\delta)\rho_2(\omega - kU_2)^2[\rho_3(\omega - kU_3)^2 - \sigma_{23}k^3]\,[\rho_1(\omega - kU_1)^2 - \sigma_{12}k^3]\\&+ \rho_2^2(\omega - kU_2)^4\,\mathrm{th}(k\delta) + \rho_2(\omega - kU_2)^2\,[\rho_3(\omega - kU_3)^2 - \sigma_{23}k^3]\\&= 0\ (\delta = b - a).\end{aligned} \tag{3.3}$$

Equation (3.3) coincides with the dispersion relation obtained in [4] for the planar case.

**Media 2 and 3 Have Identical Properties.** Let media 2 and 3 be indistinguishable ($\rho_2 = \rho_3$, $\sigma_{23} = 0$, $U_2 = U_3$). We can obviously set $U_2 = U_3 = 0$ without loss of generality. Omitting the intermediate calculations, we give the final form of the dispersion relation (2.5):

$$(\rho_1 I_{01}(a) + \rho_2 K_{01}(a))\omega^2 - 2kU_1\rho_1 I_{01}(a)\omega + \rho_1 k^2 U_1^2 I_{01}(a) - \sigma_{12}k(k^2 - a^{-2}) = 0. \tag{3.4}$$

Equation (3.4) coincides with the dispersion relation for the Rayleigh jet fragmentation problem.

**Thin Steam Film Limit.** Let b → a. We assume without loss of generality that $U_3 = 0$. Equation (2.5) then has the form

$$I_{01}(a)\rho_1(\omega - kU_1)^2 + K_{01}\rho_3\omega^2 - \sigma_\Sigma k(k^2 - a^{-2}) = 0. \tag{3.5}$$

Equation (3.5) is the dispersion relation of the stability problem for a jet of density $\rho_1$ flowing with velocity $U_1$ in a fluid of density $\rho_3$. The surface tension at the interface is $\sigma_\Sigma = \sigma_{12} + \sigma_{23}$.

**Thick Steam Film Limit.** We consider the situation a << b. Disregarding the small Bessel function groups in Eq. (2.5), we obtain

$$[I_{01}(a)\rho_1(\omega - kU_1)^2 + K_{01}(a)\rho_2(\omega - kU_2)^2 - \sigma_{12}k(k^2 - a^{-2})]\,[I_{01}(b)\rho_2(\omega - kU_2)^2 + K_{01}(b)\rho_3(\omega - kU_3)^2 - \sigma_{23}k(k^2 - b^{-2})] = 0. \tag{3.6}$$

Here the stability problem for the given system is reduced to two independent problems. The dynamics of the melt jet in a vapor medium are investigated in the first (interior) problem, in which case the first bracketed expression in (3.6) is equal to zero. The second (exterior) problem is concerned with the stability of the steam jet in the surrounding water [the second bracketed expression in (3.6) is equal to zero].

**Low-Density Steam.** If $\rho_2 << \rho_1$ and $\rho_2 << \rho_3$, terms with $\rho_2$ can be ignored in Eq. (2.5). It is then simplified and assumes the form

$$[I_{01}(a)\rho_1(\omega - kU_1)^2 - \sigma_{12}k(k^2 - a^{-2})] \times [K_{01}(b)\rho_3(\omega - kU_3)^2 - \sigma_{23}k(k^2 - b^{-2})] = 0, \tag{3.7}$$

reducing the problem to the successive stability analysis of a melt jet in vacuum and of a "vacuum jet" in water.

**4.** Disturbances occurring randomly at the initial time grow in an unstable flow regime. The most interesting species are rapidly growing disturbances, where it is important to know both their growth rate and their characteristic space scale. We use the above-derived dispersion relations to investigate this problem.

To begin with, following Epstein and Fauske [4], we estimate the thickness of the steam film surrounding the jet. We assume that steam is generated mainly in a zone situated in front of the leading edge of the water-immersed jet. The heat flux from the front end of the jet is spent in evaporating the water ahead of it:

$$\pi a^2 q = mh_{32}. \tag{4.1}$$

Here q is the heat flux from unit surface of the jet, m is the mass rate of steam generation, kg/sec, and $h_{32}$ is the latent heat of vaporization of water.

The main contribution to the heat flux is from radiation:

$$q = k_r\sigma_r T_1^4, \tag{4.2}$$

where $k_r$ is the radiating capacity of the melt, $\sigma_r$ is the Stefan–Boltzmann constant, and $T_1$ is the temperature of the melt.

The generated steam moves with velocity U relative to the jet. We write the mass balance equation for the steam:

$$m = \pi[(a + \delta)^2 - a^2]\rho_2 U \;(U = U_1 - U_2). \tag{4.3}$$

On the basis of (4.1) and (4.2) Eq. (4.3) assumes the form

$$h_{32}[(a+\delta)^2 - a^2]\rho_2 U = k_r\sigma_r T_1^4 a^2. \tag{4.4}$$

The relative steam velocity U is determined from the condition that buoyancy, pushing the steam upward, is equal to the force created by friction of the steam against the melt jet and the water. We write this condition for the part of the steam film of height $\Delta z$ as

$$(\rho_3 - \rho_2)g\Delta z\pi[(a+\delta)^2 - a^2] = F_{12} + F_{32}. \tag{4.5}$$

Here g is the free-fall acceleration, and $F_{12}$ and $F_{32}$ are the friction forces, which are functions of the steam velocity relative to the melt and the water:

$$\begin{aligned} F_{12} &= 2\pi a\Delta z c_{12}\frac{\rho_2(U_1 - U_2)^2}{2}, \\ F_{32} &= 2\pi(a+\delta)\Delta z c_{32}\frac{\rho_2|U_2|U_2}{2}. \end{aligned} \tag{4.6}$$

In Eq. (4.6) it is assumed that $U_3 = 0$, i.e., the melt jet issues into water at rest. The friction coefficients $c_{12}$ and $c_{32}$ are functions of the corresponding Reynolds numbers:

$$c_{12} = \begin{cases} 0{,}057\mathrm{Re}_{12}^{-0{,}25}, & \mathrm{Re} \geqslant 400, \\ 4\mathrm{Re}_{12}^{-1}, & \mathrm{Re} < 400, \end{cases}$$

$$\mathrm{Re}_{12} = \rho_2|U_1 - U_2|\delta/\mu_2$$

($\mu_2$ is the viscosity of the steam), which is analogous to the equation used for $c_{32}$.

Substituting the expressions for $F_{12}$ and $F_{32}$ into (4.5), we obtain

$$(\rho_3 - \rho_2)g[(a+\delta)^2 - a^2] = c_{12}\rho_2 U^2 a + c_{32}\rho_2|U_1 - U|(U_1 - U)(a+\delta). \tag{4.7}$$

The system of nonlinear equations (4.5) and (4.7) determines $\delta$ and U when the jet velocity $U_1$ is known.

This system is solved numerically by an iterative method. In the ensuing analysis, therefore, we specialize the situation. We use recently published results of experimental studies at the Argonne National Laboratory (USA) on the fragmentation of a molten aluminum jet in water [7]. In experiment No. 11 an aluminum jet with a temperature $T_1 = 973$ K was directed at a velocity $U_1 = 5$ m/sec into still water at atmospheric pressure. The radius of the jet was $a = 10$ mm, the density of the aluminum was $\rho_1 = 2700$ kg/m$^3$, and the coefficient of surface tension was $\sigma_{12} = 1$ N/m. We assume the following thermodynamic parameters for water: $\rho_2 = 0.59$ kg/m$^3$; $\rho_3 = 998$ kg/m$^3$; $h_{32} = 2.257$ MJ/kg. Solving Eqs. (4.5) and (4.7) numerically, we have $U_2 = 1.8$ m/sec and $\delta = 0.6 \cdot 10^{-4}$ m.

**5.** For these parameters we determine the size of the fastest-growing disturbances and the rate of their growth and then use this information to estimate the jet fragmentation length.

The dispersion relation (2.5) has four pairwise-conjugate complex roots. For $a << b$ (thick film approximation) Eq. (2.5) splits into two quadratic equations associated with the interior and exterior problems. Each of these equations has two complex roots. A numerical analysis has shown that all the roots can be classified into two groups for a steam film of any finite thickness: two roots and their corresponding particular solutions describe the evolution of the melt–steam interface, and the other two roots characterize the behavior of the steam–water interface. The roots of Eq. (2.5) are found by iterations using the interior and exterior solutions of the thick-film problem as initial approximations.

We transform to dimensionless variables, adopting the jet radius $a$ and the capillary wave period $t_x = (\rho_1 a^3/\sigma_{12})^{1/2}$ as characteristic scales. We denote by $x_m$ the dimensionless wave number of the fastest-growing disturbance ($x_m = k_m a$, where $k_m$ is the corresponding dimensioned wave number). Let $t_m$ be a dimensionless time constant characterizing the growth of the fastest-growing disturbances:

$$t_m = [|\mathrm{Im}\,\omega_m|t_x]^{-1}$$

($\omega_m$ is the complex frequency of these disturbances).

For the specific situation discussed above, where the film thickness corresponds to the dimensionless parameter $\beta = b/a = 1.006$, the numerical solution of the dispersion relation (2.5) gives $t_m^{-1} = 1.703$ and $x_m = 1.48$.

These data can be used to estimate the jet fragmentation length. According to [8], the jet fragmentation time is of the order of magnitude

$$t_b = k_m a t_m t_x,$$

and the length over which the jet disintegrates is

$$L = U_1 t_b = U_1 k_m a t_m t_x.$$

The corresponding dimensionless quantity has the form

$$\frac{L}{2a} = \frac{1}{2}\sqrt{\mathrm{We}}\,x_m t_m \quad (\mathrm{We} = \rho_1 U_1^2 a/\sigma_{12}) \tag{5.1}$$

(We is the Weber number).

Substituting the values of $x_m$ and $t_m$ obtained from the dispersion relation into Eq. (5.1), we obtain $L/2a = 11.16$, which is in good agreement with the experimental value $L/2a = 14$. It is interesting to note that the application of the asymptotic dispersion relation (3.5) for a thin steam film gives $L/2a = 3.03$. Consequently, despite the small thickness of the steam film, its inclusion is of fundamental importance. The film, in effect, shields the melt jet from disturbances arriving from the water.

To ascertain the influence of the thickness of the steam film on the stability of the melt in water, we have carried out a series of computations with different values of $\beta$; the results are given in Table 1, from which it follows that the presence of even a thin steam film significantly stabilizes the melt jet, and the water ceases to influence the jet dynamics altogether at a mere 10% film thickness, when the thick-film approximation can be used. In this regard, the above-discussed case of maximally rarefied steam (3.7), when the jet is actually in contact with vacuum, yields a dimensionless fragmentation length of the jet $L/2a = 26.10$.

The thickness of the film in the cited experiment lies in the transition range, where the dependence of the jet fragmentation length on the film thickness is significant. Consequently, it is important here to have a reliable estimate of the film thickness. The estimate obtained in the present study yields good agreement with experiment. The estimate given in [4], on the other hand, makes the film thickness too large and, as a result, produces an appreciable discrepancy with experiment.

Gabor et al. [7] have reported two more experiments to determine the fragmentation length of a molten aluminum jet in water (experiments No. 9 and No. 10). The results of the calculations for these experiments are given in Table 2.

A comparison of our investigation with a study [4] of the analogous problem in a planar setting has shown that the analysis of stability on the basis of the planar formulation for the above-indicated jet parameters leads to patently erroneous results for $a < 5$ cm. The planar formulation becomes valid for a jet of radius $a > 50$ cm.

Consequently, a comparison with the experimental results leads to the conclusion that the proposed model of the fragmentation of a melt jet in water, based on an analysis of its hydrodynamic stability, adequately describes the process and can be used to estimate the parameters of a melt–water mixture in studying the consequences of nuclear power plant accidents with core meltdown.

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